Math Assignment Unit 6

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Course: MATH 1201-01 - AY2026-T1

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Task 1

Point A has coordinates $\left(-\frac{\sqrt{3}}{2}, \frac{1}{2}\right)$ on the unit circle after one revolution.

(i) Calculate all 6 values of the trigonometric functions.

On the unit circle, for any point , the trigonometric functions are defined as:

- $\sin(\theta) = y$
- $\cos(\theta) = x$
- $\tan(\theta) = \frac{y}{x}$, provided $x \neq 0$
- $\csc(\theta) = \frac{1}{y}$, provided $y \neq 0$
- $\sec(\theta) = \frac{1}{x}$, provided $x \neq 0$
- $\cot(\theta) = \frac{x}{y}$, provided $y \neq 0$

Given $x = -\frac{\sqrt{3}}{2}$, $y = \frac{1}{2}$:

1. $\sin(\theta) = y = \frac{1}{2}$
2. $\cos(\theta) = x = -\frac{\sqrt{3}}{2}$
3. $\tan(\theta) = \frac{y}{x} = \frac{1/2}{-\sqrt{3}/2} = -\frac{1}{\sqrt{3}} = -\frac{\sqrt{3}}{3}$
4. $\csc(\theta) = \frac{1}{y} = \frac{1}{1/2} = 2$
5. $\sec(\theta) = \frac{1}{x} = \frac{1}{-\sqrt{3}/2} = -\frac{2}{\sqrt{3}} = -\frac{2\sqrt{3}}{3}$
6. $\cot(\theta) = \frac{x}{y} = \frac{-\sqrt{3}/2}{1/2} = -\sqrt{3}$

(ii) Determine the quadrant in which the point lies and provide the reason.

The point $\left(-\frac{\sqrt{3}}{2}, \frac{1}{2}\right)$ has a negative x-coordinate and a positive y-coordinate. This places it in Quadrant II.

Reason: In the Cartesian coordinate system, Quadrant II is defined by x<0 and y>0.

(iii) Calculate the angle formed by point A and the reference angle with the positive X-axis.

The reference angle is the acute angle between the terminal side of the angle and the x-axis. Since the point is in Quadrant II, the reference angle θ_{ref} is calculated using the absolute value of the cosine or sine function.

We know that $\cos(\theta) = -\frac{\sqrt{3}}{2}$ and $\sin(\theta) = \frac{1}{2}$. The reference angle corresponds to the angle whose cosine is $\frac{\sqrt{3}}{2}$ and sine is $\frac{1}{2}$, which is 30° or $\frac{\pi}{6}$ radians.

Since the point is in Quadrant II, the actual angle θ from the positive x-axis is:

$$\theta = 180^\circ - 30^\circ = 150^\circ \quad \text{or} \quad \theta = \pi - \frac{\pi}{6} = \frac{5\pi}{6} \text{ radians}$$

Thus, the angle formed by point A with the positive x-axis is $\frac{5\pi}{6}$ radians (or 150°), and the reference angle is $\frac{\pi}{6}$ radians (or 30°).

Task 2

Alice observes an object at the top of a 6-meter tree. Initially, she is 4 meters away (Point A), then moves 2 meters closer to 2 meters away (Point B).

(i) Find the angles formed by Alice at points A and B relative to the top of the tree. What are these angles called?

These angles are called angles of elevation — the angle between the horizontal line of sight and the line of sight up to an object.

Let θ_A be the angle at Point A (4 meters away), and θ_B be the angle at Point B (2 meters away).

Using right triangle trigonometry:

At Point A:

$$\tan(\theta_A) = \frac{\text{opposite}}{\text{adjacent}} = \frac{6}{4} = 1.5$$

$$\theta_A = \tan^{-1}(1.5) \approx 56.31^\circ$$

At Point B:

$$\tan(\theta_B) = \frac{6}{2} = 3$$

$$\theta_B = \tan^{-1}(3) \approx 71.57^\circ$$

So, the angles of elevation are approximately 56.31° at Point A and 71.57° at Point B.

(ii) Determine whether angle A is larger than angle B. Make a conclusion about the comparison of angles when observing an object from a distance versus close.

Angle A (56.31°) is smaller than Angle B (71.57°). Therefore, angle A is not larger than angle B.

Conclusion: As the observer moves closer to the object (from 4m to 2m), the angle of elevation increases. This means that objects appear “higher” or more steeply angled when viewed from closer distances.

(iii) Find the distances between the object and points A and B.

These are the hypotenuses of the right triangles formed.

At Point A (distance from Alice to object):

$$\text{Distance}_A = \sqrt{4^2 + 6^2} = \sqrt{16 + 36} = \sqrt{52} = 2\sqrt{13} \approx 7.21 \text{ meters}$$

At Point B (distance from Alice to object):

$$\text{Distance}_B = \sqrt{2^2 + 6^2} = \sqrt{4 + 36} = \sqrt{40} = 2\sqrt{10} \approx 6.32 \text{ meters}$$

Task 3

(a)Write the equation of a sine or cosine function with midline = 5, amplitude = 13, period =2 π , and phase shift = 0.

General form of a sinusoidal function:

$$y = A\sin(B(x - C)) + D \quad \text{or} \quad y = A\cos(B(x - C)) + D$$

Where:

- A = amplitude
- $B = \frac{2\pi}{\text{period}}$
- C = phase shift
- D = midline

Given:

- Amplitude $A = 13$
- Period = $2\pi \rightarrow B = \frac{2\pi}{2\pi} = 1$
- Phase shift $C = 0$
- Midline $D = 5$

Equation (using sine for example):

$$y = 13\sin(x) + 5$$

(b) For the trigonometric equation $y = 15\tan\left(\frac{\pi x}{3} + 2\right)$, find its stretching factor, period, phase shift, vertical asymptotes, and domain.

Standard form: $y = A\tan(Bx + C) + D$

Here, $A = 15$, $B = \frac{\pi}{3}$, $C = 2$, $D = 0$

- **Stretching factor (vertical stretch):** $|A| = 15$
- **Period:** $\frac{\pi}{|B|} = \frac{\pi}{\pi/3} = 3$
- **Phase shift:** Solve $Bx + C = 0 \rightarrow \frac{\pi}{3}x + 2 = 0 \rightarrow x = -\frac{6}{\pi}$
  So, phase shift is $-\frac{6}{\pi}$ (to the left)
- **Vertical asymptotes:** Occur where the argument of tan equals $\frac{\pi}{2} + n\pi$:

$$\frac{\pi x}{3} + 2 = \frac{\pi}{2} + n\pi$$

$$\frac{\pi x}{3} = \frac{\pi}{2} - 2 + n\pi$$

$$x = \frac{3}{\pi}\left(\frac{\pi}{2} - 2 + n\pi\right) = \frac{3}{2} - \frac{6}{\pi} + 3n$$

So vertical asymptotes are at $x = \frac{3}{2} - \frac{6}{\pi} + 3n$, for integer n.

- **Domain:** All real numbers except where vertical asymptotes occur.

Domain: $x \in \mathbb{R} \setminus \left\{\frac{3}{2} - \frac{6}{\pi} + 3n \mid n \in \mathbb{Z}\right\}$

(c) Find the coordinates of points a, b, c, d, e, and f on the given graph.

The graph is a sine wave with amplitude 1 and period 360° (or 2π radians), centered on the x-axis (midline = 0). It starts at a maximum at $\theta = 0^\circ$, so it is a **cosine function**: $y = \cos(\theta)$.

From the graph:

- Point a: At $\theta = -360^\circ$, same as $\theta = 0^\circ \rightarrow y = \cos(0^\circ) = 1 \rightarrow (-360^\circ, 1)$
- Point b: At $\theta = -90^\circ$, $y = \cos(-90^\circ) = 0 \rightarrow (-90^\circ, 0)$
- Point c: At $\theta = -180^\circ$, $y = \cos(-180^\circ) = -1 \rightarrow (-180^\circ, -1)$
- Point d: At $\theta = 90^\circ$, $y = \cos(90^\circ) = 0 \rightarrow (90^\circ, 0)$
- Point e: At $\theta = 180^\circ$, $y = \cos(180^\circ) = -1 \rightarrow (180^\circ, -1)$
- Point f: At $\theta = 270^\circ$, $y = \cos(270^\circ) = 0 \rightarrow (270^\circ, 0)$

Note: The graph shows point c at -180° and point e at 180°, both at y = -1, which matches cosine.

Task 4

(i) Fill the table with a preferred trigonometric function Y = f(X) and its inverse $f^{-1}(Y)$.

We choose the **sine function**, $f(X) = \sin(X)$, over the interval $[-\frac{\pi}{2}, \frac{\pi}{2}]$ to ensure it's one-to-one and invertible.

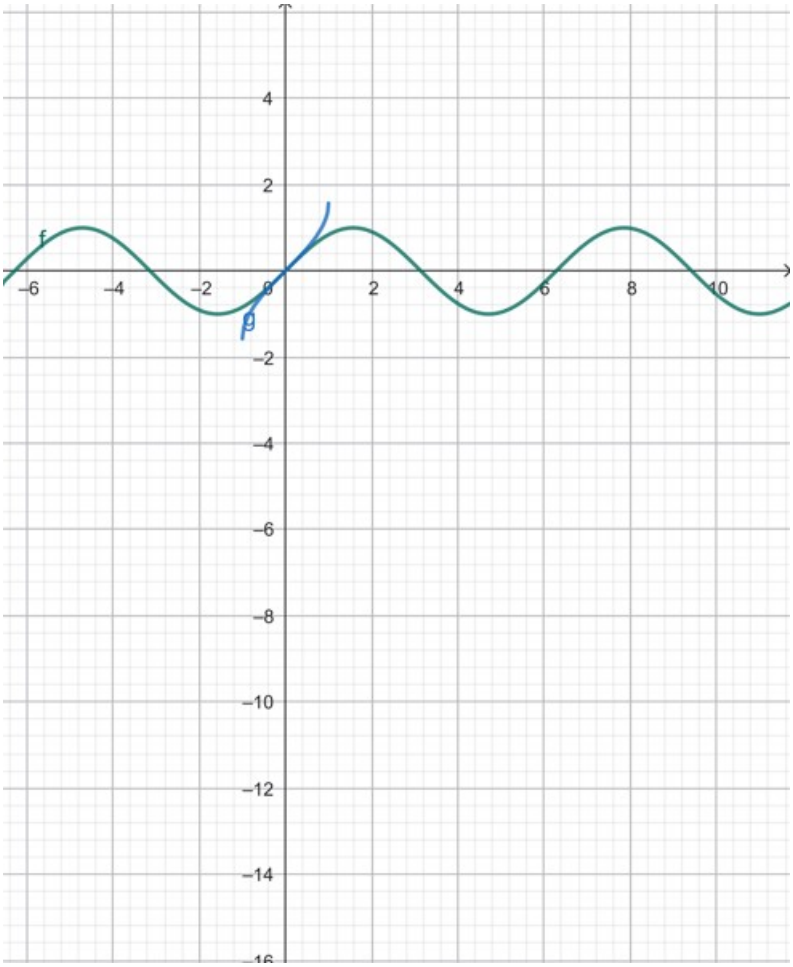
Inverse: $f^{-1}(Y) = \arcsin(Y)$

X	0	$\frac{\Pi}{3}$	$\frac{2\Pi}{3}$	$\frac{\Pi}{2}$	Π	$\frac{4\Pi}{3}$	2Π
Y = f(X) = sin(X)	0	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{3}}{2}$	1	0	$-\frac{\sqrt{3}}{2}$	0
f^{-1}(Y) = arcsin(Y)	0	$\frac{\pi}{3}$	Not defined*	$\frac{\pi}{2}$	Not defined*	Not defined*	Not defined*

*Note: The inverse sine function, $\arcsin(Y)$, is only defined for $Y \in [-1, 1]$, and returns values in $[-\frac{\pi}{2}, \frac{\pi}{2}]$. Therefore, for $X = \frac{2\pi}{3}$, $Y = \sin(\frac{2\pi}{3}) = \frac{\sqrt{3}}{2}$, but $\arcsin(\frac{\sqrt{3}}{2}) = \frac{\pi}{3}$, not $\frac{2\pi}{3}$. So technically, we cannot define $f^{-1}(Y)$ for all X beyond the principal range. To maintain a true inverse, we restrict X to $[-\frac{\pi}{2}, \frac{\pi}{2}]$. For this table, we show the values as computed, noting the limitation.

Alternatively, to avoid confusion, we can restrict the domain of X to $[-\frac{\pi}{2}, \frac{\pi}{2}]$, but since the table includes up to 2π, we'll proceed with the understanding that $f^{-1}(Y)$ only gives the principal value.

(ii) Utilize the graphing tool to depict both functions Y = f(X) and f^{-1}(Y) on the same graph.



(iii) Observe the tabular values and the graphs of Y = f(X) and $f^{-1}(Y)$, answer the following:

a. Examine the periodicity of the chosen trigonometric function, Y = f(X), as presented in the table.

The sine function is periodic with period 2π. From the table, we observe:

- $\sin(0) = 0$, $\sin(2\pi) = 0$
- $\sin(\frac{\pi}{3}) = \sin(\frac{2\pi}{3}) = \frac{\sqrt{3}}{2}$ (but note: $\frac{2\pi}{3} = \pi - \frac{\pi}{3}$, showing symmetry, not periodicity directly)
- $\sin(\pi) = 0$, $\sin(2\pi) = 0$

The function repeats every 2π , confirming periodicity.

b. Discuss the domain and range of both Y = f(X) and $f^{-1}(Y)$.

- For $f(X) = \sin(X)$:
  - Domain: All real numbers, $\mathbb{R}$
  - Range: $[-1, 1]$
- For $f^{-1}(Y) = \arcsin(Y)$:
  - Domain: $[-1, 1]$ (since sine only outputs values in this interval)
  - Range: $[-\frac{\pi}{2}, \frac{\pi}{2}]$ (principal values)

c. Determine whether the function Y = f(X) demonstrates even, odd, or neither.

Check if $f(-x) = f(x)$ (even) or $f(-x) = -f(x)$ (odd).

For $f(x) = \sin(x)$:

$$\sin(-x) = -\sin(x)$$

Therefore, sine is an **odd function**.

References

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